Improved ambiguity resolution by regional differential modelling of the ionosphere

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Biography

Lambert Wanninger received his Dipl.-Ing. and his Dr.-Ing. in Geodesy in 1988 and 1994 from the Universität Hannover, Germany. He has several years experience in various aspects of precise GPS applications especially in the field of ionospheric effects on GPS and in ionospheric research with GPS. He is currently employed as research associate at the Geodätisches Institut, Technische Universität Dresden, Germany.

Abstract

Fast and on-the-fly ambiguity resolution algorithms are limited to short baselines mainly due to ionospheric refraction. Even under the moderate and mostly undisturbed ionospheric conditions of the mid-latitudes, the differential errors often exceed 1 ppm of the baseline length. Much larger errors are found in the presence of ionospheric disturbances and in the equatorial region.

In order to reduce these differential errors and thus to extend the use of fast and on-the-fly ambiguity resolution from short to medium-length baselines, a differential ionospheric model was developed whose parameters are derived from dual-frequency phase observations of at least three GPS monitor stations. Differential ionospheric corrections are produced epoch-by-epoch and satellite-by-satellite for any other GPS station in the area.

Examples are presented which demonstrate the improvement in ambiguity resolution under undisturbed ionospheric conditions in the mid-latitudes, and also in the presence of ionospheric disturbances (medium-scale Traveling Ionospheric Disturbances). These disturbances are able to prevent fast and on-the-fly ambiguity resolution even on short baselines (≤ 10 km). The differential ionospheric model removes most of these disturbing effects.

Introduction

Resolution of the double difference carrier phase ambiguities is the key to precise (cm accuracy) baseline coordinates from GPS measurements. In recent years, fast ambiguity resolution algorithms and ambiguity resolution on-the-fly were introduced (e.g. *Hatch 1990, Frei and Beutler 1990*). These algorithms find the correct set of double difference ambiguities after just several seconds or few minutes of observations. They usually produce reliable results if no large observation errors affect the phase and code observations. Thus, code noise needs to be on a low level in order to provide a good initial position estimation from code observations. Multipath effects have to be minimized by antenna design and by careful attention to antenna siting. Moreover, distance-dependent errors have to be kept small by restriction to short baselines. In fact, these distance-dependent errors limit fast ambiguity resolution to baselines with a maximum length of a few kilometers.

Two kinds of distance-dependent errors exist: ionospheric refraction and orbit errors. Even under Selective Availability (SA) broadcast orbit errors have seldom exceeded 10 m. Applying the well known but pessimistic rule-of-thumb that a 20 m orbit error results in 1 ppm baseline error shows that orbit errors contribute to the overall error budget in maximum in the order of 0.5 ppm of the baseline length. The effects of ionospheric errors, however, are often much larger (1 – 2 ppm and more) even under the moderate ionospheric conditions of the mid-latitudes and even at the present minimum of the sunspot cycle. Larger errors occur in the presence of ionospheric disturbances, in the equatorial region, and during years of high sunspot activity. Incidents have been observed where the L_1 ionospheric baseline error exceeded 10 ppm of the baseline length (*Wanninger 1993a,b*).

In order to illustrate the contributions of orbit errors and ionospheric errors to double difference phase observables, a 24 hour data set of a 44 km baseline has been arbitrarily selected from our database of permanent GPS observations in Germany. After ambiguity fixing, the double difference range residuals have been plotted for L_1 and for the ionosphere-free linear combination L_0 (Fig. 1). Ionospheric refraction affects the L_1-signal only,

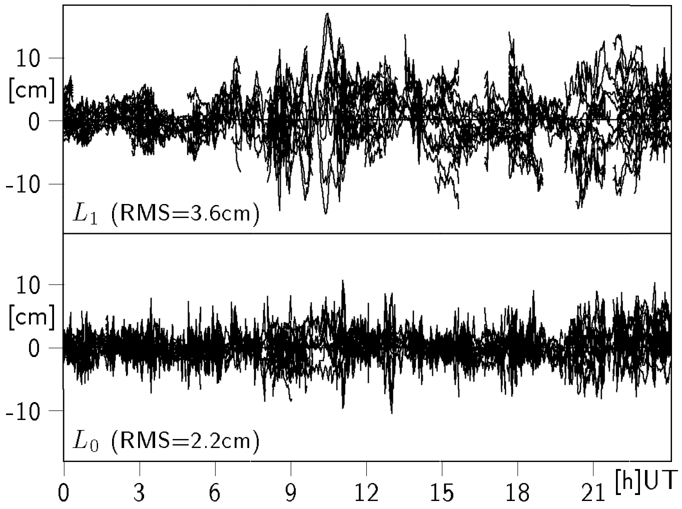

Figure 1: Double difference range residuals in all satellite combinations of a 44 km baseline, for L_1 and the ionosphere-free linear combination L_0, observed in Germany on September 7, 1994.

whereas both signals contain orbit errors and all other kinds of errors. Hence, the difference between the two plots are mainly due to the ionosphere (some differences can be attributed to the higher observation noise of the L_0-signal and to multipath). Whereas small differences can be detected between midnight and sunrise (low ionospheric electron content), large differences are found during daylight hours (high electron content). L_2 observations and linear combinations like widelane and narrowlane experience similar ionospheric effects as L_1. The difference in RMS-values (Fig. 1) illustrates that ionospheric refraction is the main error source in medium-length baselines and that ionospheric refraction causes the main difficulty for ambiguity resolution.

One approach to extend the use of fast ambiguity resolution algorithms to medium-length baselines (10 – 50 km) consists in the reduction of ionospheric errors by application of appropriate ionospheric models. In recent years, various publications dealt with models of the absolute total electron content (TEC) derived from dual-frequency GPS observations of one or several receivers (Fig. 2a). These models were used to reduce single-frequency baseline coordinate errors (*Georgiadou and Kleusberg 1988, Wild et al. 1989*) and to improve ambiguity resolution on long baselines and for observation sessions of several hours (*Mervart et al. 1994*). Such a model consists of one set of coefficients for all satellites and an observation session of several hours. Their main limitation lies in the inability to reproduce small-scale or medium-scale structures of the ionospheric electron content.

The idea of differential ionospheric modelling was in-

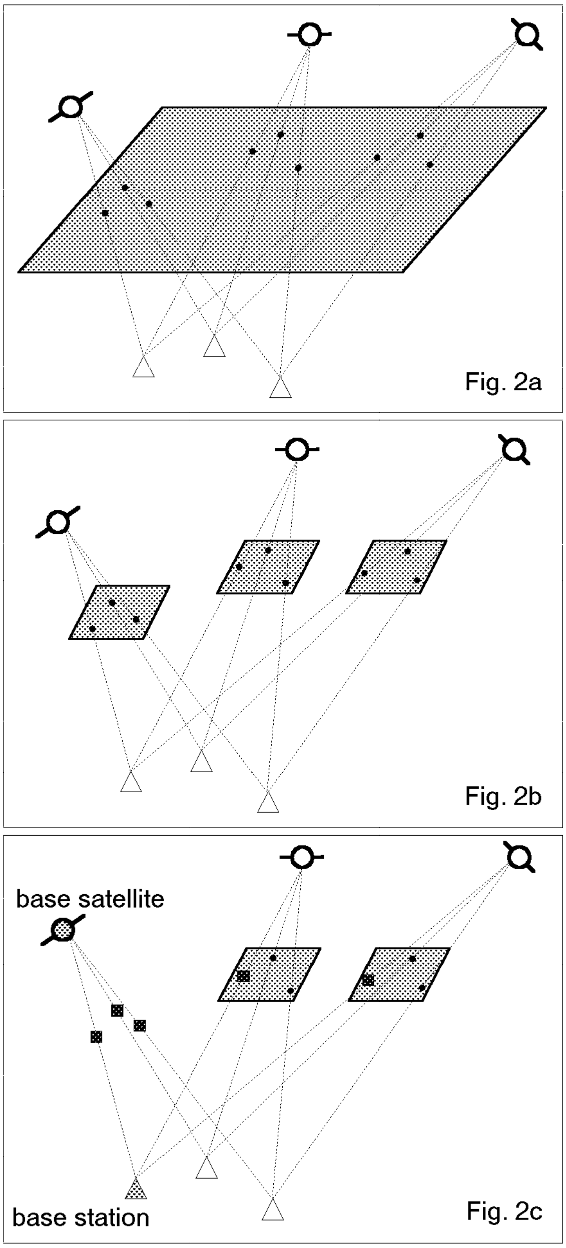

Figure 2: Different levels of ionospheric models for differential GPS (simplified concepts). 2a: Modelling of the absolute TEC. 2b: Differential modelling of the ionosphere. 2c: Differential modelling of the ionosphere using double differences.

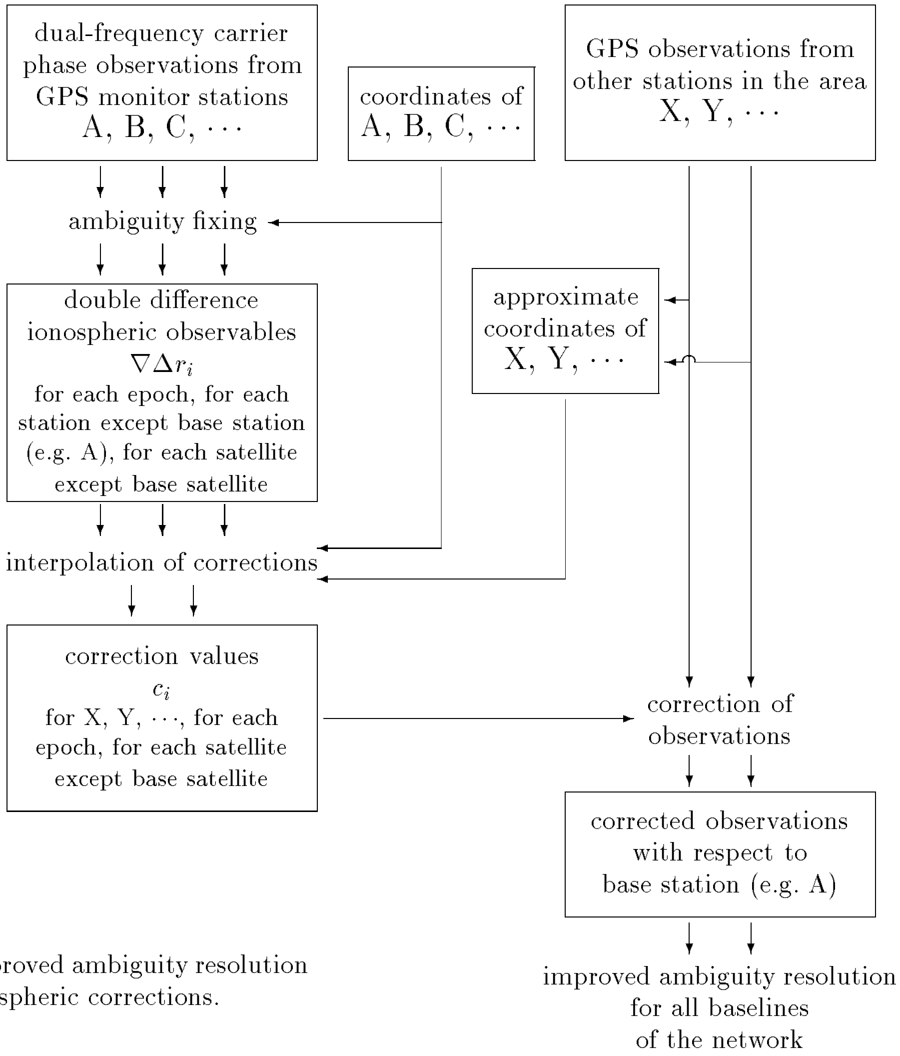

Figure 3:
The concept of improved ambiguity resolution by differential ionospheric corrections.

troduced by *Webster and Kleusberg (1992)*. Their model provides epoch-by-epoch and satellite-by-satellite ionospheric corrections. The ionospheric delays of a station equipped with a single-frequency receiver are estimated from interpolation of ionospheric delay observations of three surrounding monitor stations using the intersection points of the GPS signal paths with an ionospheric single-layer model at a height of 350 km (Fig. 2b). The problem of ambiguities in the ionospheric delays derived from dual-frequency phase data is overcome by assuming that the ambiguity differences between the monitor stations are, on average, equal to zero.

In this research, dual-frequency carrier phase ambiguities are resolved and fixed in the network of monitor stations, thus yielding differential ionospheric delays in the most accurate GPS mode. Correction values are then determined on the level of double differences epoch-by-epoch and satellite-by-satellite from interpolation of ionospheric delay values of the monitor stations using their coordinates and approximate coordinates of the new station (Fig. 2c). Thus, in contrast to *Webster and Kleusberg (1992)* our approach requires complete ambiguity fixing between monitor stations. Moreover, the interpolation algorithm has been simplified.

Regional Differential Ionospheric Model

The regional differential ionospheric model is derived from dual-frequency phase data of at least three GPS monitor stations surrounding the area of interest. It is based on the double difference observables of the ionospheric linear combination L_i of the phase observations.

In a first processing step, ambiguity resolution and fixing has to be performed for the network of monitor stations (compare Fig. 3, see also *Wanninger 1995*). Although their distances may be of the order of 50 km, ambiguity resolution is simplified because the baseline coordinates are known, dual-frequency receivers are employed and long observation periods can be used. If the modelling is to be accomplished in real-time or almost in real-time, ambiguity resolution in the network of monitor stations must be performed and checked continuously. The reliable fixing of the double difference ambiguities of a newly risen satellite may require several minutes, half an hour, or an even longer period of observation data. As long as the ambiguities of a particular satellite could not be fixed, no ionospheric corrections can be applied to phase observations of this satellite.

After ambiguity fixing, we obtain unambiguous double difference ionospheric observables $\nabla\Delta r_i$ for each epoch. They refer to a base station and a base satellite. In each observation epoch the ionospheric model consists of at least two (number of monitor stations minus one) $\nabla\Delta r_i$ values per visible satellite. Ionospheric corrections c_i for any station in the area can than be computed by interpolation using the known latitude and longitude coordinates of the monitor stations and the approximate latitude and longitude coordinates of the stations to be determined. The correction values are interpolated epoch by epoch, i.e. in every observation epoch an independent set of corrections is produced.

If three monitor stations are available, the interpolation of correction values is performed by a linear interpolation algorithm. If more monitor stations are available, either the best group of three stations (for example the three closest surrounding stations) should be selected or several correction values from sets of three stations can be computed and averaged or a more sophisticated interpolation algorithm could be used. The main advantage of more than three monitor stations lies in the ability to determine correction values even if only some (but at least three) monitor stations including the base station could provide observations.

In a further processing step, the ionospheric correction values are scaled to ionospheric effects on L_1 and L_2 in order to yield corrections for the original phase observations or also for the code observations. Corrections can be applied to the observations of any station in the area including the monitor stations. The observations of the base station and of the base satellite need not to be modified because their correction values are zero.

Since double difference corrections are applied to non-differenced observations, a further baseline processing must only be performed between stations whose observations have been manipulated by correction values based on identical error models. Manipulated data must not be combined with original observations with the exception of the observations of the base station.

These ionospheric corrections have effects on single-frequency and dual-frequency ambiguity resolution and on single-frequency coordinate results. Ionosphere-free coordinate solutions are not affected. Their results remain unchanged.

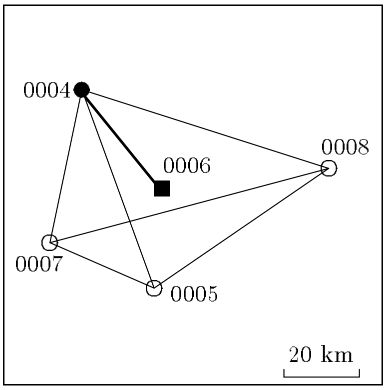

Figure 4: Network of GPS monitor stations and the 34 km baseline 0004–0006.

In an operational mode, ambiguity resolution in a regional network of several monitor stations surrounding a number of new stations consists of the following processing steps: (a) ambiguity resolution in the network of monitor stations, (b) estimation of ionospheric corrections, (c) modification of the observations of all stations with the exception of the base station, and (d) improved ambiguity resolution for all baselines in the network.

Undisturbed Ionospheric Conditions

Observations were selected arbitrarily from the data sets of the already existing dense network of permanent GPS stations in North Germany (Figure 4). 13 hours of Trimble SST/SSE observations (June 17, 1994) were used to test the described algorithm. The analysis of the ionospheric conditions from dual-frequency phase observations revealed that almost undisturbed and thus average mid-latitude ionospheric conditions were present. The GEONAP software package (*Wübbena 1989*) was used to perform ambiguity resolution in the network of monitor stations (0004,0005,0007,0008).

Ionospheric corrections were predicted for station 0006 from the dual-frequency phase observations of at least three surrounding permanent tracking stations. When dual-frequency observations of all four surrounding stations existed, the mean of two predicted values (triangle 0004-0007-0008 and triangle 0004-0005-0008) was taken as correction value for the observation of station 0006. In both triangles, station 0004 was used as base station, therefore the observations of 0004 needed not to be corrected in order to perform improved ambiguity resolution for the baseline 0004-0006.

The successful correction of ionospheric effects is shown by comparison of double difference phase residuals of L_1 and the widelane linear combination L_W for

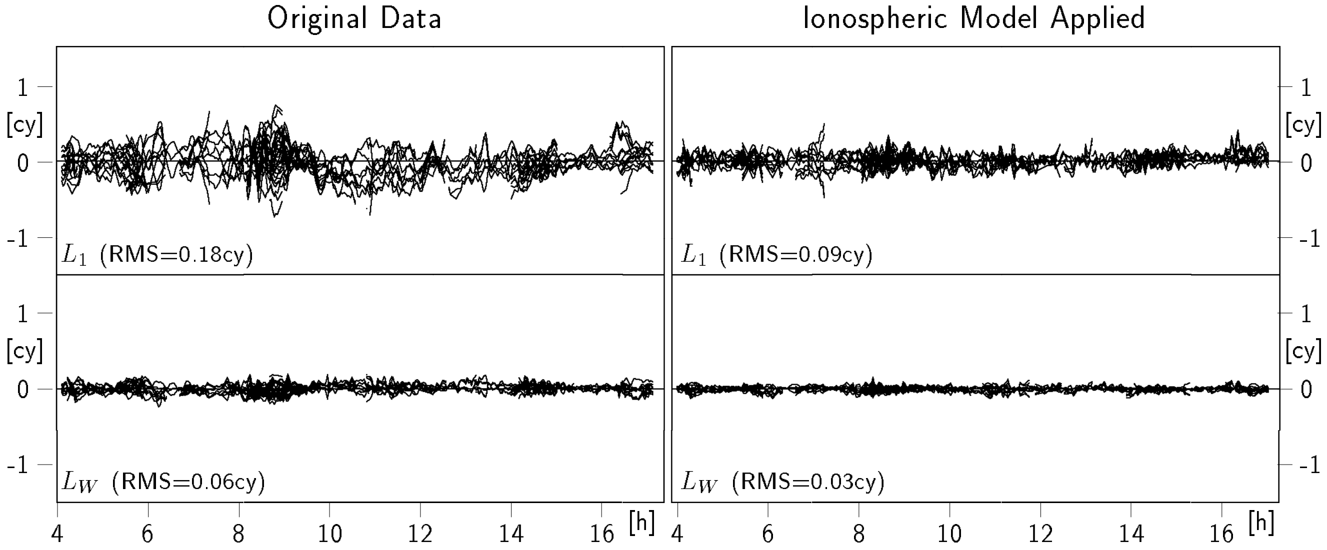

Figure 5: Double difference phase residuals after ambiguity fixing in all satellite combinations, in L_1 and widelane L_W, for the original data and with ionospheric model applied, 34 km baseline of Figure 4, observed under undisturbed ionospheric conditions.

	original observations	ionospheric model applied
correct ambiguities fixed	45 (87%)	50 (96%)
incorrect ambiguities fixed	7 (13%)	2 (4%)

Table 1: Ambiguity fixing with GPSurvey, 15-min blocks of observations, 34 km baseline of Figure 4, undisturbed ionospheric conditions.

the original data and also for the corrected data (Fig. 5). The RMS of all double difference phase observations improved in L_1 from 0.18 cy to 0.09 cy and in L_W from 0.06 cy to 0.03 cy. Systematic effects of ionospheric refraction which can be identified by constantly large residuals over some period of time were considerably reduced. Whereas 95% of the L_W residuals were smaller than 9.9 cm before ionospheric correction, afterwards 95% of the L_W-residuals were smaller than 6.5 cm. But the corresponding figures for L_1 show no improvement (original data: 95% smaller than 7.0 cm, corrected data: 95% smaller than 7.2 cm). The ionospheric corrections considerably improved the L_1-RMS value, but nevertheless the number of large residuals could not be reduced. Most probably, they are caused by L_1-specific but non-ionospheric errors.

In order to verify whether the ionospheric corrections improve ambiguity resolution, 15-min blocks of observations were processed with a non-scientific manufacturer's software package (Trimble's GPSurvey 2.0). The GPS observations were loaded from RINEX-format, on the one hand the original observations and on the other hand RINEX phase and code observations after ionospheric correction. In 13% of the 15-min blocks, the ambiguity resolution algorithm selected an incorrect set of double difference ambiguities with the original data (the coordinate errors, obtained by comparison of the 15-min baseline solution with the results of the complete data set, considerably exceeded the test limits of 5 cm in the horizontal components or 8 cm in the vertical component). After application of the ionospheric correction this percentage was reduced to 4 (Table 1). The remaining failures are attributed to poor receiver-satellite-geometry causing large coordinate errors in the initial coordinate solution. Hence, with the exception of outages due to poor geometry, the ionospheric model guaranteed correct ambiguity resolution of a 34 km baseline, with 15-min blocks of observations and with a standard software package.

Medium-Size Ionospheric Disturbances

The most common ionospheric disturbances in mid-latitude regions are caused by medium-scale travelling ionospheric disturbances (MSTIDs). They mainly occur during daylight hours in winter months in years of maximum solar activity. They complicate ambiguity resolution even on baselines shorter than 10 km. Single-frequency coordinate errors can exceed 10 ppm of the baseline length (*Wanninger 1993a*).

An example of strong MSTIDs was found in a 9-hour

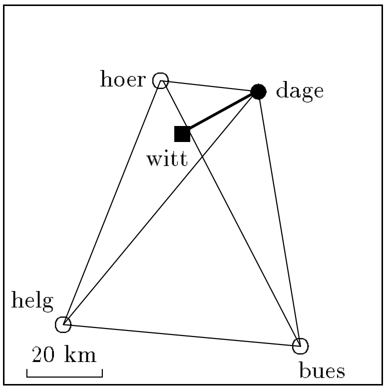

Figure 6:
Network of GPS monitor stations and the 23 km baseline dage–witt.

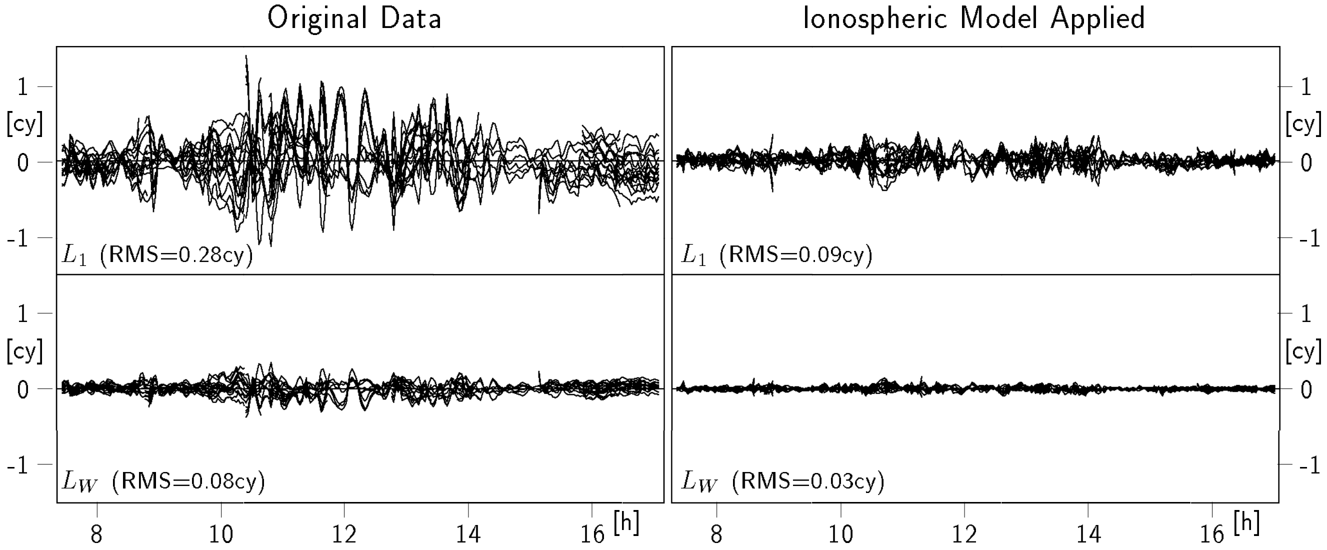

Figure 7: Double difference phase residuals after ambiguity fixing in all satellite combinations, in L_1 and widelane L_W, for the original data and with ionospheric model applied, 23 km baseline of Figure 6, observed during a period of ionospheric disturbances (medium-scale TIDs).

data set observed with Trimble SSE in North Germany on March 16, 1993 (Fig. 6). The analysis of the ionospheric conditions from dual-frequency phase observations revealed large and periodically changing ionospheric effects with typical MSTID-periods of 10 to 20 minutes. These effects can also be observed in the L_1 and L_W double difference phase residuals (Fig. 7). L_1 double difference phase residuals show periodic disturbances with amplitudes up to 1 cycle and periods of 15 to 30 minutes. Here, only epoch-by-epoch and satellite-by-satellite ionospheric modelling can produce accurate corrections.

The comparison of double difference phase residuals in L_1 and in L_W for the original data and for the corrected data demonstrates the successful application of the differential ionospheric model (Fig. 7). The RMS of all double difference phase observations improved in L_1 from 0.28 cy to 0.09 cy and in L_W from 0.08 cy to 0.03 cy. Whereas 95% of the L_1 residuals were smaller than 11,0 cm (L_W: 14,0 cm) before applying ionospheric corrections, afterwards 95% of the L_1-residuals were smaller than 3.5 cm (L_W: 4.9 cm). Consequently, the disturbing effects of MSTIDs could greatly be reduced.

In order to verify the improvement in ambiguity resolution, GPSurvey was used again to process 15-min blocks of observations of the baseline dage–witt. Whereas ambiguity resolution failed in 24% using the original data, the ambiguities of all 38 15-min blocks could correctly be solved after applying ionospheric corrections (Table 2). Hence, the ionospheric model guaranteed correct ambiguity fixing even under disturbed ionospheric conditions.

Moreover, the 23 km static baseline has been processed in kinematic mode and an ambiguity-fixed widelane solution has been produced, i.e. independent sets of coordinates have been calculated for every epoch using the wide-

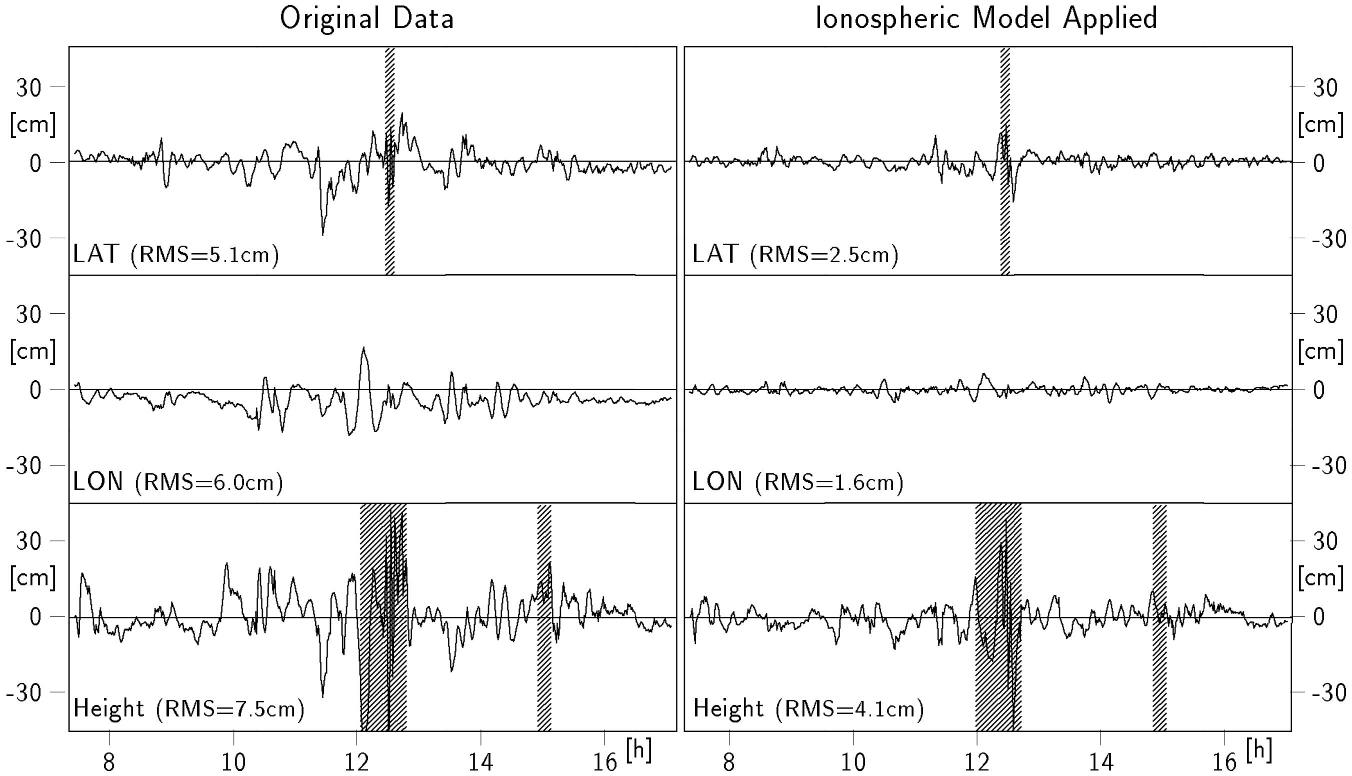

Figure 8: Coordinate errors of a kinematic processing (widelane solution) of the 23 km static baseline of Figure 6, observed during a period of ionospheric disturbances (medium-scale TIDs), shaded areas indicate periods with DOP ≥ 7.

	original observations	ionospheric model applied
correct ambiguities fixed	29 (76%)	38 (100%)
incorrect ambiguities fixed	9 (24%)	0 (0%)

Table 2: Ambiguity fixing with GPSurvey, 15-min blocks of observations, 23 km baseline of Figure 6, observed during a period of ionospheric disturbances (medium-scale TIDs).

lane linear combination. Two kinds of ionospheric effects can be distinguished (Fig. 8, original data), medium-scale TIDs causing errors with amplitudes up to 20 cm and with periods of 15 to 30 minutes, and errors constant over several hours due to the absolute ionospheric electron content (see e.g. the fairly constant error of -6 cm in longitude between 1400 and 1700 hours). Ionospheric corrections completely removed the latter effects and considerably reduced the effects of MSTIDs. The RMS of the coordinate errors improved from 5–6 cm to 1.5–2.5 cm in the horizontal components and from 7.5 to about 4 cm in the vertical component.

Conclusion

Regional differential modelling of the ionosphere based on dual-frequency observations of at least three GPS monitor stations can successfully be applied to reduce ionospheric effects on medium-length baselines and thus improve ambiguity resolution. Fast and on-the-fly ambiguity resolution can now be extended from short ($\leq$ 10 km) to medium-length (10 – 50 km) baselines, thus reducing the required minimum observation time.

The requirement of complete dual-frequency ambiguity fixing in the network of monitor stations limits this approach to networks with baseline lengths of less than 50 to 100 km. If the modelling is to be performed in (near-) real-time ambiguity resolution in the network of monitor stations must be performed and checked continuously.

Despite those limitations, these regional ionospheric error models enhance differential GPS in medium-sized networks. Most of the ionospheric effects are removed even under ionospheric disturbed conditions. Ambiguity resolution is improved for all those techniques which rely on small ionospheric effects.

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